

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2025

First Semester

Civil Engineering

(Common to All Branches)

24BSMA101 - MATRICES AND CALCULUS

Regulations - 2024

Duration: 3 Hours

Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

- | | Marks | K-Level | CO |
|---|-------|---------|-----|
| 1. The Cayley-Hamilton theorem states that every square matrix satisfies its _____
(a) Characteristic equation (b) Minimal polynomial
(c) Inverse matrix (d) Eigenvector equation | 1 | K1 | CO1 |
| 2. The determinant of an orthogonal matrix is always:
(a) 0 (b) 1 or -1 (c) Positive (d) Negative | 1 | K1 | CO1 |
| 3. If $x = r \cos \theta$, $y = r \sin \theta$ then the value of $\frac{\partial(x,y)}{\partial(r,\theta)}$ is equal to
(a) θ (b) $-\theta$ (c) r (d) $-r$ | 1 | K1 | CO2 |
| 4. For function $f(x, y)$ to have minimum value at (a, b) value is?
(a) $rt - s^2 > 0$ and $r < 0$ (b) $rt - s^2 > 0$ and $r > 0$
(c) $rt - s^2 < 0$ and $r < 0$ (d) $rt - s^2 < 0$ and $r > 0$ | 1 | K1 | CO2 |
| 5. If $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$, what is the value of $\nabla \cdot \vec{F}$.
(a) $x+y+z$ (b) 3 (c) 1 (d) 0 | 1 | K2 | CO3 |
| 6. A vector field with zero curl is called _____
(a) Solenoidal (b) Irrotational (c) Conservative (d) Both b and c | 1 | K1 | CO3 |
| 7. Green's theorem relates _____
(a) A line integral around a simple closed curve C to a double integral over the region R it encloses
(b) A surface integral to a volume integral
(c) A curl to a divergence
(d) None of the above | 1 | K1 | CO4 |
| 8. Stokes' theorem relates _____
(a) A surface integral of curl F over surface S to the line integral of F along the boundary curve C
(b) A volume integral to a surface integral
(c) Gradient to divergence
(d) None of the above | 1 | K1 | CO4 |
| 9. What is the area enclosed by the curve $r = 2 \cos \theta$ in polar coordinates?
(a) π (b) $\pi/2$ (c) 2π (d) 4π | 1 | K2 | CO5 |
| 10. The convergence of the Fourier series is guaranteed by
(a) Dirichlet's conditions (b) Mean value theorem
(c) Fundamental theorem of calculus (d) Fourier Integral theorem | 1 | K1 | CO6 |

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

- | | | | |
|---|---|----|-----|
| 11. The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & 2 & 2 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue of A. | 2 | K2 | CO1 |
| 12. Find the nature of quadratic form $2xy + 2yz + 2zx$. | 2 | K2 | CO1 |

13. Find. $\lim_{(x,y) \rightarrow (0,0)} \left[\frac{x^2 y}{x^4 + y^2} \right]$ 2 K2 CO2
14. If $x = r \cos \theta, y = r \sin \theta$, then find $\frac{\partial(x,y)}{\partial(r,\theta)}$. 2 K2 CO2
15. Prove that $F = (4x + z^2)\vec{i} + (6x^2 + z)\vec{j} + (3yz^2 + y)\vec{k}$ is solenoidal. 2 K2 CO3
16. Find the constant a such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal. 2 K2 CO3
17. Find the area of the circle of radius ' a ' using Green's theorem. 2 K2 CO4
18. State Stoke's theorem. 2 K1 CO4
19. Evaluate $\int_0^1 \int_1^2 x(x + y) dy dx$. 2 K2 CO5
20. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\sin \theta} r d\theta dr$. 2 K2 CO5
21. Find half range sine series for $f(x) = k$ in $0 < x < \pi$. 2 K2 CO6
22. State the Dirichlet's conditions for the existence of the Fourier expansion of $f(x)$, in the interval $(0, 2\pi)$. 2 K1 CO6

PART - C ($6 \times 11 = 66$ Marks)

Answer ALL Questions

23. a) Using Cayley-Hamilton theorem find A^4 and A^{-1} when 11 K3 CO1
- $$A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$
- OR**
- b) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ into canonical form by an orthogonal transformation. 11 K3 CO1
24. a) Expand $e^x \cos y$ in powers of x & y as far as the terms of the third degree using Taylor's series method. 11 K3 CO2
- OR**
- b) The temperature $u(x, y, z)$ at any point in space is $u = 400xyz^2$. Find the highest temperature on surface of the sphere $x^2 + y^2 + z^2 = 1$. 11 K3 CO2
25. a) Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational and find its scalar potential. 11 K3 CO3
- OR**
- b) Find a and b so that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at the point $(1, -1, 2)$. 11 K3 CO3
26. a) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. 11 K3 CO4
- OR**
- b) Verify Gauss Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelopiped bounded by $x = 0, y = 0, z = 0$ and $x = a, y = b, z = c$. 11 K3 CO4
27. a) Evaluate by changing the order of integration $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy dx$. 11 K3 CO5

OR

b) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integrals. 11 K3 CO5

28. a) If $f(x) = \frac{1}{2}(\pi - x)$, find the Fourier series of period 2π in the interval $(0, 2\pi)$. Hence 11 K3 CO6
deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.

OR

b) Find the Fourier series of the function $f(x) = x^2$ in the interval $-\pi < x < \pi$. Hence 11 K3 CO6
deduce that using Parseval's theorem $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots = \frac{\pi^4}{90}$.