Reg. No.																

Question Paper Code	13579

B.E. / B.Tech. - DEGREE EXAMINATIONS, APRIL / MAY 2025

Fourth Semester

Artificial Intelligence and Data Science

(Common to Computer Science and Engineering (AIML))

20BSMA404 - LINEAR ALGEBRA AND ITS APPLICATIONS

Duration: 3 Hours | Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

	Question	*Marks*	*K – Level*	*CO*
1.	If $v_1 = (1, 2)$ and $v_2 = (3, 4)$, what is the linear combination of $2v_1 + 3v_2$ (a) (5, 10) (b) (7, 10) (c) (11, 18) (d) (8, 14)	*1*	*K2*	*CO1*
2.	In LU Decomposition, which matrix is represented as 'L' (a) The diagonal matrix (b) The identity matrix (c) The lower triangular matrix (d) The upper triangular matrix	*1*	*K1*	*CO1*
3.	Choose the linearly dependent set of vectors from the following (a) (2, 1), (4,-1) (-2, 2) (b) (2,-3, 6), (4, 6, 12) (c) (3, 2) ,(6, 2) (d) (-4, 3), (8,6)	*1*	*K2*	*CO2*
4.	Find the value of m such that the vector $(m, 7, -4)$ is a linear combination of vectors $(-2\ ,2,\ 1)$ and $(2,\ 1, -2)$. (a) 2 (b) -2 (c) 0 (d) -1	*1*	*K2*	*CO2*
5.	Let V and W be a vector spaces, and let $T\colon V \to W$ be linear. If V is finite dimensional, then, nullity(T) + rank(T)=dim(V) is known as (a) Replacement Theorem (b) Dimension theorem (c) Nullity Theorem (d) Rank Theorem	*1*	*K1*	*CO3*
6.	Let $T\colon V \to W$ be a mapping and if $N(T) = \{0\}$, then, T is (a) linear (b) one-one (c) onto (d) none of the above	*1*	*K1*	*CO3*
7.	In a real inner product space, The inner product is denoted by (a) (u, v) (b) u·v (c) [u, v] (d) ⟨u, v⟩	*1*	*K1*	*CO4*
8.	The primary purpose of the Gram-Schmidt process in linear algebra is _______ (a) To find a basis of a vector space (b) To orthogonalize a set of vectors (c) To determine the rank of a matrix (d) To compute eigenvalues of a matrix	*1*	*K1*	*CO4*
9.	SVD stand for ____________ (a) Singular Value Decomposition (b) Standard Value Determination (c) Systematic Value Decomposition (d) Singular Value Distribution	*1*	*K1*	*CO5*
10.	The primary purpose of linear algebra in data science is _____________ (a) To visualize data in three dimensions (b) To analyze complex algorithms (c) To model and solve systems of equations (d) To compute statistical measures of data	*1*	*K1*	*CO5*

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

	Question	*Marks*	*K – Level*	*CO*
11.	Find a system of equations that is equivalent to the given vector equation $x_1 \begin{bmatrix} -2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 8 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.	*2*	*K2*	*CO1*
12.	Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$.	*2*	*K2*	*CO1*
13.	Find whether the given system is consistent or inconsistent: $3x + 2y = 4;\ 6x + 4y = 10$	*2*	*K2*	*CO1*
14.	Define subspace.	*2*	*K1*	*CO2*
15.	Determine whether the following set is linearly dependent or linearly independent. $\{x^3 + 2x^2, -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\}$ in $P_3(R)$.	*2*	*K2*	*CO2*

K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze; K5 – Evaluate; K6 – Create

16.	Define a basis of a vector space V. Give an example.	2	K1	CO2
17.	Let T:$R^2 \rightarrow R^2$defined by $T(a_{1,}a_2)$ = ($2a_1$+a_2 ,a_1). Show that T is linear.	2	K2	CO3
18.	Let T:$R^3 \rightarrow R^2$defined by T($a_{1,}a_2,a_3$) = ($a_1 - a_2$,$2a_3$). Find N(T).	2	K2	CO3
19.	Define inner product.	2	K1	CO4
20.	Consider the following polynomials $g(t) = 3t - 2$ in P(t) with the inner product $\langle f,g\rangle = \int_0^1 f(t)g(t)dt$. Find $\| g \|$.	2	K2	CO4
21.	Define Principal component analysis.	2	K1	CO5
22.	What is total variance?	2	K1	CO5

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23.	a) (i)	Solve the system of linear equations by Cramer's rule $2x + 2y - z = 8,\ -3x - y + 2z = -11,\ -2x + y + 2z = -3$	6	K3	CO1
	(ii)	Solve the following system of equations using LU decomposition**:** $x + y = 5,\ 2x + 3y = 11.$	5	K3	CO1
		OR			
	b)	Solve the following system of equations using Gaussian elimination $x + y + z = 6, \quad x + 2y + 3z = 14, \quad 2x + y + z = 9.$	11	K3	CO1
24.	a)	Show that $\mathbb{R}^n = \{(x_1, x_2, x_3, \cdots, x_n): x_i \in \mathbb{R}\}$ is a vector space over F with respect to addition and scalar multiplication defined component wise.	11	K3	CO2
		OR			
	b)	Let V be a vector space and $\beta = \{u_1, u_2, u_3, \ldots, u_n\}$ be the sub set of V. Then β is the basis for V if and only if each $v \in V$can be uniquely expressed as a linear combination of vectors of β. that is $v = a_1u_1 + a_2u_2 + a_3u_3 + \cdots, +a_nu_n$ for unique scalars $a_1, a_2, a_3, \ldots, a_n$.	11	K3	CO2
25.	a)	Prove that there exists a linear transformation T:$R^2 \rightarrow R^3$ such that T (1,1) = (1,0,2) and T (2,3) = (1, -1,4). Find T (8,11).	11	K3	CO3
		OR			
	b)	Let T be linear operator on P_2(R) defined by $T(f(x)) = f(1) + f'(0)x + \left(f'(0) + f''(0)\right)x^2$. Test T for diagonalizability, and if, finds the basis for β for V such that $[T]_\beta$ is a diagonal matrix.	11	K3	CO3
26.	a) (i)	Find a least – squares solution of the inconsistent system Ax = b for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.	6	K3	CO4
	(ii)	Let $W = Span\{X_1, X_2\}$, Where $X_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ $and\ X_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Construct an orthogonal basis $\{v_1, v_2\}$ for W.	5	K3	CO4
		OR			
	b)	Determine a QR factorization of A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.	11	K3	CO4
27.	a)	For the covariance matrix $\sum = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, determine the principal components and the proportion of variance accounted by the principal components.	11	K3	CO5

OR

b) Compute the singular value decomposition of $A=\begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. *11 K3 CO5*

28. a) (i) State and prove Cauchy-Schwarz inequality. *6 K3 CO4*

(ii) Determine the matrix U such that $A = U\sum V^T$, where A=$\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$. *5 K3 CO5*

OR

b) (i) Show that $\{u_1, u_2, u_3\}$ is an orthogonal set, where $u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} \frac{-1}{2} \\ -2 \\ \frac{7}{2} \end{bmatrix}$ *6 K3 CO4*

(ii) Explain how linear algebra is used in principal Component Analysis and its important in data science. *5 K3 CO5*