

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2025

Third Semester

Artificial Intelligence and Data Science

20BSMA302 - PROBABILITY AND STATISTICAL MODELING

Regulations - 2020

(Use of Statistical table is permitted)

Duration: 3 Hours

Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

	Marks	K- Level	CO
1. In a Poisson distribution, $E[X]$ is (a) npq (b) np (c) λ (d) λ^2	1	K1	CO1
2. Poisson distribution is a limiting case of (a) Negative binomial distribution (b) Binomial distribution (c) Geometric distribution (d) Exponential distribution	1	K1	CO1
3. The parameters of bivariate normal distribution are (a) μ, ρ (b) μ, γ (c) μ, σ (d) x, s	1	K1	CO2
4. If X and Y are independent variables then $\text{Cov}(X, Y)$ is (a) 0 (b) 1 (c) -1 (d) 2	1	K1	CO2
5. The statistical constants of population are called (a) Statistic (b) Variables (c) Constants (d) Parameter	1	K1	CO3
6. If the calculated value is less than the table value in testing a hypothesis we (a) Reject null hypothesis (b) Accept null hypothesis (c) Accept alternative hypothesis (d) None of the above	1	K1	CO3
7. Which test is used to check the randomness of samples? (a) The Kolmogorov – Smirnov test (b) The Kruskal-Wallis test (c) The Mann-Whitney test (d) Run test	1	K1	CO4
8. Non parametric statistics are also called (a) Distribution free statistics (b) Analysis free statistics (c) Mean free statistics (d) None of these	1	K1	CO4
9. The type of estimation approach uses sample data to compute a single values for a population parameter is (a) Point estimation (b) Data estimation (c) Interval estimation (d) Statistical estimation	1	K1	CO5
10. A time series has (a) 2 components (b) 3 components (c) 4 components (d) 5 components	1	K1	CO5

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

11. Find C if $P(X = n) = C \left(\frac{2}{3}\right)^n$, $n = 1, 2, 3 \dots$	2	K2	CO1
12. If the random variable X has the MGF $M_X(t) = \frac{3}{3-t}$, then find the standard deviation of X.	2	K2	CO1
13. A fair die is rolled independently 10 times. Find the probability that the faces 1 to 6 occur the following respective number of times: 2, 1, 3, 1, 2, 1.	2	K2	CO2
14. State Central limit theorem.	2	K2	CO2
15. Define Type-I and Type-II error.	2	K1	CO3
16. Write any two uses of t-distribution.	2	K2	CO3
17. Give the test statistic used in Kendall test of concordance.	2	K2	CO4

18. Explain sign test. 2 K2 CO4
 19. Define point estimation. 2 K1 CO5
 20. Define Maximum likelihood estimator. 2 K1 CO5
 21. Find the value of k if $f(x, y) = kx(1-x), 0 < x, y < 1$ is to be a joint density function. 2 K2 CO1
 22. What are the advantages of non-parametric methods of testing hypothesis? 2 K1 CO3

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23. a) A Random Variable X has the following probability function 11 K3 CO1

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k ²	2k ²	7k ² +k

Find (i) k (ii) $P(1.5 < X < 4.5 | X > 2)$ (iii) distribution function of 'X'
 iv) smallest value of 'n' for which $P(X \leq n) > 1/2$.

OR

- b) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, What is the probability that at least 1 of them would have scored above 75? 11 K3 CO1

24. a) If X and Y are random variables having the joint density function 11 K3 CO2

$$f(x, y) = \frac{1}{8}(6 - x - y); 0 < x < 2; 2 < y < 4$$

$$= 0, \text{ otherwise}$$

i) Find $P(X < 1)$ ii) Find the covariance of 'X' and 'Y'.

OR

- b) A sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using central limit theorem, find the probability with which the mean of the sample will not differ from 60 by more than 4. 11 K3 CO2

25. a) The mean height and the standard deviation of height of eight randomly chosen soldiers are 166.9 cm and 8.29 cm respectively. The corresponding values of six randomly chosen sailors are 170.3 cm and 8.50 cm respectively. Based on this data, can we conclude that soldiers are, in general, shorter than sailors? 11 K3 CO3

OR

- b) The accompanying data resulted from an experiment comparing the degree of soiling for fabric copolymerized with the 3 different mixtures of met acrylic acid. Analyze the classification. 11 K3 CO3

Mixture 1: 0.56 1.12 0.90 1.07 0.94
 Mixture 2: 0.72 0.69 0.87 0.78 0.91
 Mixture 3: 0.62 1.08 1.07 0.99 0.93

26. a) In 30 tosses of a coin, the following sequence of head (H) and tails (T) is obtained 11 K3 CO4
 HTTHTHHHTHTTHTHTHTHTTHTHTHT

(a) Determine the number of runs
 (b) Test at 5% level of significance, whether the sequence is random.

OR

- b) Two methods of instruction to apprentices are to be evaluated. A director assigns 15 randomly selected trainees to each of the two methods. Due to drop outs, 14 complete in Batch1 and 12 complete in Batch2. An achievement test was given to these successful candidates. Their scores are as follows. 11 K3 CO4

Method I	70	90	82	64	86	77	84	79	82	89	73	81	83	66
Method II	86	78	90	82	65	87	80	88	95	85	76	94		

Test whether the two methods have significant difference in effectiveness. Use Mann-Whitney test at 5% significance level.

27. a) Find the Maximum Likelihood estimator for θ in Binomial Distribution with parameter n, θ . 11 K3 CO5

OR

- b) Show that the auto correlation function of the second-order MA process 11 K3 CO5

$$X_t = Z_t + 0.7Z_{t-1} - 0.2Z_{t-2}$$

is given by

$$\rho(k) = \begin{cases} 1 & \text{if } k = 0 \\ 0.37 & \text{if } k = \pm 1 \\ -0.13 & \text{if } k = \pm 2 \\ 0 & \text{otherwise} \end{cases}$$

28. a) (i) Trains arrive at a station at 15 minutes intervals starting at 4.am. If a passenger arrive at a station at a time that is a uniformly distributed between 9.00 and 9.30. Find the probability that he has to wait for a train for i) less than 6 minutes ii) more than 10 minutes. 6 K3 CO1

- (ii) Let X_1, X_2, \dots, X_n be independent identically distributed random variables with mean $\mu = 2$ and variance $\sigma^2 = 1/4$. Use the Central limit theorem to estimate $P(192 < S_n < 210)$ where $S_n = X_1 + X_2 + \dots + X_{100}$. 5 K3 CO2

OR

- b) (i) Determine the moment generating function of Poisson distribution. 6 K3 CO1
- (ii) Suppose we have a bowl with 10 marbles, 2 red marbles, 3 green marbles, and 5 blue marbles. We randomly select 4 marbles from the bowl, with replacement. What is the probability of selecting 2 green marbles and 2 blue marbles? 5 K3 CO2