Reg. No.																

Question Paper Code	14262

B.E. / B.Tech. - DEGREE EXAMINATIONS, NOV / DEC 2025

Fourth Semester

Artificial Intelligence and Data Science

(Common to Computer Science and Engineering (AIML))

20BSMA404 - LINEAR ALGEBRA AND ITS APPLICATIONS

Regulations - 2020

Duration: 3 Hours | Max. Marks: 100

PART - A (MCQ) (10 × 1 = 10 Marks)

Answer ALL Questions

No.	Question	*Marks*	*K – Level*	*CO*
1.	What is the rank of null matrix? (a) 1 (b) 2 (c) n (d) 0	1	K1	CO1
2.	Find the value of y using Cramer's rule $3x + y + 2z = 3;\ 2x - 3y - z = -3;\ x + 2y + z = 4$ (a)1 (b) -1 (c) 2 (d) -2	1	K2	CO1
3.	What is the dimension of R^n matrix is (a) $n+1$ (b) $n-1$ (c) n (d) 0	1	K1	CO2
4.	Any subset of a linearly independent set is (a) lineally dependent (b) lineally independent (c) basis (d) none of the above	1	K2	CO2
5.	If $T: U \to V$ is a linear transformation with dim(U) = 4 and nullity(T) = 2, then rank of T is (a) 1 (b) 2 (c) 0 (d) 4	1	K2	CO3
6.	If A and B are square matrices of the same order, then trace(AB) = (a) trace (A + B) (b) trace (A) trace (B) (c) trace (BA) (d) trace (A) + trace (B)	1	K2	CO3
7.	Two vectors u and v of an inner product space are orthogonal if their inner product is (a) Equal to 0 (b) Equal to u.v (c) less than 1 (d) Equal to 1	1	K2	CO4
8.	In an inner product space, if u is orthogonal to v then $\|u+v\|^2$ is equal to (a) $\|u\|^2 - \|v\|^2$ (b) $\|u\|^2$ (c) $\|v\|^2$ (d) $\|u\|^2 + \|v\|^2$	1	K2	CO4
9.	SVD stand for –-------------------- (a) Singular Value Decomposition (b) Standard Value Determination (c) Systematic Value Decomposition (d) Singular value Distribution	1	K1	CO5
10.	What is the primary goal of Principal Component Analysis? (a) Classification (b) Clustering (c) Dimensionality reduction (d) Regression	1	K1	CO5

PART - B (12 × 2 = 24 Marks)

Answer ALL Questions

No.	Question	*Marks*	*K – Level*	*CO*
11.	Define Rank of a Matrix.	2	K1	CO1
12.	Solve the system using Cramer's rule: $2x + 3y = 12$, $4x - y = 5$.	2	K2	CO1
13.	Let $W = \{(a_1, a_2, a_3) / a_1 = a_3 + 2\}$.Prove that W is not a subspace of V.	2	K2	CO2
14.	Find the value of m such that the vector (m, 7, 4) is a linear combination of vectors (2,2,1) and (2,1 ,2).	2	K2	CO2
15.	Prove that the identity element of a Vector Space is unique.	2	K2	CO2
16.	Let T:$R^2 \to R^2$defined by T($a_{1,} a_2$) = ($2a_1$+a_2 ,a_1) .Show that T is linear.	2	K2	CO3
17.	Define Range Space and Null Space.	2	K1	CO3
18.	If $T: V \to V$ is a linear transformation defined by $T(v) = v, \forall\ v \epsilon V$, then check whether T is one-one?	2	K2	CO3
19.	Show that $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$ for $x, y \in V$.	2	K2	CO4
20.	Define inner product of a vector space.	2	K1	CO4
21.	Define Singular Value.	2	K1	CO5

22. Define Principal component analysis. *2* *K1* *CO5*

PART - C (6 × 11 = 66 Marks)

Answer ALL Questions

23.	a)	Solve the system of equation $x+y+z=1, 3x+y+z=5; x-2y-5z=10$ by LU decomposition method.	*11*	*K3*	*CO1*
		OR			
	b)	Solve the equations $3x+2y+7z=4;\quad 2x+3y+z=5; 3x+4y+z=7$ by Gauss Elimination method.	*11*	*K3*	*CO1*
24.	a)	If $P(F)$ denotes the set of all polynomials over a field F, then prove that $P(F)$ is a vector space over $P(F) = \{a_0 + a_1x + a_2x^2 + \cdots + a_nx^n\}$ where $a_i \in F$.	*11*	*K3*	*CO2*
		OR			
	b) (i)	Prove that the intersection of two subspaces of a vector space is a subspace.	*6*	*K3*	*CO2*
	(ii)	Determine whether u and v are linear dependent or not if $u = (-4,6,-2)$ and $v = (2,-3,1)$.	*5*	*K3*	*CO2*
25.	a)	Prove that there exists linear transformation $T: R^2 \to R^3$ such that $T(1,1)=(1,0,2)$ *and* $T(2,3)=(1,-1,4)$ Find $T(8,11)$.	*11*	*K3*	*CO3*
		OR			
	b)	State and prove Dimension Theorem.	*11*	*K3*	*CO3*
26.	a)	Prove that R^2 is an inner product space with an inner product defined by $\langle x,y\rangle = a_1b_1 - a_2b_1 - a_1b_2 + 2a_2b_2$.	*11*	*K3*	*CO4*
		OR			
	b)	Let V be the set off all polynomials of degree $\leq$ 2 together with zero polynomial. V is a real inner product space with the inner product defined by $\langle f(x), g(x)\rangle = \int_{-1}^{1} f(x)\,g(x)\,dx.$ Starting with the basis $\{1, x, x^2\}$, find an orthonormal basis of V by Gram-schmidt process.	*11*	*K3*	*CO4*
27.	a)	Find the singular value decomposition method $\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$.	*11*	*K3*	*CO5*
		OR			
	b)	Let $X = (X_1, X_2, X_3)$ be a three-dimensional random vector with the following covariance matrix $\sum = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Also, determine the principal components and the proportion of variance accounted by the principal components.	*11*	*K3*	*CO5*
28.	a)	Determine QR decomposition of a matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$.	*11*	*K3*	*CO4*
		OR			
	b)	Apply Gram-Schmidht process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $B\{v_1, v_2, v_3\}$ where $v_1 = (1,0,1); v_2 = (1,3,1);$ & $v_3 = (3,2,1)$.	*11*	*K3*	*CO4*